Analysis and Design of Wide-Band Matched Waveguide Bends including Discontinuities

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Abstract— **Waveguide bends with small radii of curvature are realized by using properly selected discontinuities which, placed inside the curve, allow low return losses to be achieved over large band-widths. The component is designed by using an efficient computer code which employs the *local modes* approach to analyze curved sections, while discontinuities are rigorously accounted for by considering their accessible modes. Theoretical simulations are compared with experimental results showing very good accuracy.**

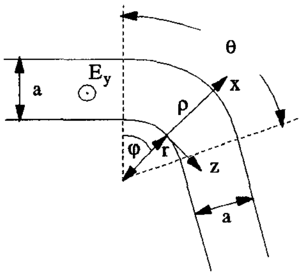

Fig. 1. H-plane bend showing used coordinate systems.

I. Introduction

Waveguide bends are crucial for sophisticated microwave systems such as radar seekers, satellite beam forming networks, etc. [1], [2], [3], where, in order to minimize space requirements, it is often required to realize curves with short radii, but nevertheless exhibiting low return loss over a wide-band. Wide-band matched (WBM) bends were first introduced by de Ronde [4] by inserting suitable matching elements (ME) such as stubs, notches, etc., on an experimental basis.

While a considerable amount of literature addresses the full-wave analysis of bends [5]–[9], there is a lack of information on the design of compact WBM bends. Moreover, most current approaches do not coexist favourably with possibly the commonest method used for analizing interacting discontinuities. This method, in fact, is based on the use of the Generalised Scattering Matrices (GSM), considering all the accessible modes relative to each discontinuity. It would be appropriate, therefore, to characterize the bend too in terms of its GSM. As an example, numerically oriented methods, although useful for analysis purposes, fail to provide insight on how to select suitable ME, while their limited numerical efficiency prevents from their use in the optimization routines necessary to design WBM components.

In this contribution we present several new solutions to design compact WBM bends, both in the E- and H-planes, which are easy to manufacture, avoiding the use of trimming elements and providing a fairly robust design both from the electrical and mechanical viewpoint. In addition, we also introduce for the first time the combination of the local modes concept [10] and of that of accessible modes, that is suitable to account for the presence of discontinuities and ME inside the bend. Thanks to its numerical efficiency, this approach is an ideal candidate for CAD purposes.

The analysis by the local modes approach is described in the next section, while section III. illustrates the proposed WBM bends; finally, in section IV. the theoretical simulations are compared with experimental data as well as with other theoretical results.

II. Analysis by Local Modes

Let us consider a rectangular waveguide H-plane bend of angle θ, as shown in a schematic top view in Fig. 1, with incident field the fundamental TE_{10} mode. This field has no variation in the y-direction and the only fields components are E_y, H_x, H_z. The method of local modes describes the field in each section of the bend by means of a *superposition of modes of the locally straight waveguide.* In this way the electric field in the bend is expressed in the following manner

$$E_y(x,z) = \sum_{n=1}^{\infty} V_n(z)\phi_n(x) \qquad (1)$$

while the magnetic field is given by

$$H_x(x,z) = -\sum_{n=1}^{\infty} I_n(z)\phi_n(x) \qquad (2)$$

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In practice, sums are truncated after few terms, say N, since the field inside the bend is quite similar to that of straight sections. However, while the modes in a straight section are independent of each other, in the curved sections they couple and their propagation inside the bend is described by the following system of equations

$$\partial_\varphi V_m = -A_m I_m - \sum_{n=1}^{N} B_{mn} I_n \quad (3)$$

$$\partial_\varphi I_m = -C_m V_m - \sum_{n=1}^{N} D_{mn} V_n \quad (4)$$

where the coefficients A_m, B_m, C_{mn}, D_{mn}, have the simple closed-form expression derived hereafter.

A. Coefficients of the Telegrapher's equation

In each section of the bend the field may be obtained from a potential ψ solution of the Helmholtz' equation expressed in cylindrical coordinates, i.e. of

$$\left(\partial_\rho^2 + \frac{1}{\rho}\partial_\rho\right)\psi + \frac{1}{\rho^2}\partial_\varphi^2\psi + k^2\psi = 0 \quad (5)$$

from which the transverse field components are obtained, in cylindrical coordinates, as

$$\begin{aligned} E_y &= j\omega\mu\psi \\ H_\rho &= \frac{1}{\rho}\partial_\varphi\psi \end{aligned} \quad (6)$$

In order to find the actual form of the field we make use of the expansion (1) and (2). Note that in these expansions the $\phi_n(x)$ constitute a complete, orthonormal, basis, but they are *not* the modal basis for our curved section. In fact, the $\phi_n(x)$ are the modal function for the *straight* waveguide; as such they satisfy the following wave equation

$$\partial_x^2\phi_n + k^2\phi_n = \beta_n^2\phi_n \quad (7)$$

where β_n is the propagation constant.

With reference to Fig. 1, the relationship between the cylindrical and the rectangular coordinate system is provided by

$$\begin{aligned} \partial_z &= \frac{1}{\rho}\partial_\varphi \\ \rho &= r + x \\ \partial_x &= \partial_\rho \end{aligned} \quad (8)$$

By substituting the expansion (1) and (2) into the equations (6) and (5) it is possible to obtain the two generalized telegrapher's equation linking voltages and currents along the bend as described in the following.

1) First generalized telegrapher's equation: By substituting (1) and (2) into (6), noting that $H_x = H_\rho$, yields

$$\sum_{n=1}^{N} \partial_\varphi V_n\phi_n = -j\omega\mu\rho\sum_{n=1}^{N} I_n\phi_n \quad (9)$$

Taking advantage of the orthonormality of the basis functions ϕ_n, (9) provides the first of the generalized telegrapher's equations, i.e.

$$\partial_\varphi V_m = -j\omega\mu r I_m - j\omega\mu\sum_{n=1}^{N} H_{mn} I_n \quad (10)$$

where we have introduced the following coupling terms

$$H_{mn} = <\phi_m, x\phi_n> \quad (11)$$

Equation (10) clearly shows the mode coupling that takes place in the bend. In particular, for H-plane bends the coupling terms H_{mn} are given by

$$H_{mn} = \frac{2}{a}\int_0^a x\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{a}x\right)dx = \begin{cases} \frac{a}{2} - \frac{a}{\pi^2(m+n)^2}\left[\cos(m+n)\pi - 1\right] & m = n \\ \frac{a}{\pi^2(m-n)^2}\left[\cos(m-n)\pi - 1\right] - \\ \quad \frac{a}{\pi^2(m+n)^2}\left[\cos(m+n)\pi - 1\right] & m \neq n \end{cases} \quad (12)$$

2) Second generalized telegrapher's equation: In order to derive the second generalized telegrapher's equation it is expedient to note that, by using (8) the first term in (5) may be written as

$$\left(\partial_\rho^2 + \frac{1}{\rho}\partial_\rho\right)\psi = \partial_x^2\psi + \frac{1}{(r+x)}\partial_x\psi \quad (13)$$

while the second term in (5), by using (6), yelds

$$\frac{1}{\rho^2}\partial_\varphi^2\psi = \frac{1}{\rho}\partial_\varphi H_\rho = -\frac{1}{\rho}\sum_{n=1}^{N}\partial_\varphi I_n(\varphi)\phi_n(x) \quad (14)$$

After inserting the two above equations into (5) and by using (7), we get

$$\sum_{n=1}^{N}(r+x)\beta_n^2 V_n\phi_n + \sum_{n=1}^{N} V_n\frac{\partial\phi_n}{\partial x} = j\omega\mu\sum_{n=1}^{N}\frac{\partial I_n}{\partial\varphi}\phi_n \quad (15)$$

Taking advantage of mode orthonormality, one finally obtains

$$\frac{\partial I_m}{\partial\varphi} = -j\frac{\beta_m^2 r}{\omega\mu}V_m - j\sum_{n=1}^{N}\left(\frac{\beta_n^2}{\omega\mu}H_{mn} + \frac{f_{mn}}{\omega\mu}\right)V_n \quad (16)$$

where the following coupling integral has been introduced

$$f_{mn} = <\phi_m, \frac{\partial\phi_n}{\partial x}> = \frac{2}{a}\frac{n\pi}{a}\int_0^a \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{a}x\right)dx = \begin{cases} 0 & m = n \\ \frac{n}{a}\left\{\frac{\left[1-(-1)^{m+n}\right](m-n)+\left[1-(-1)^{m-n}\right](m+n)}{m^2-n^2}\right\} & m \neq n \end{cases} \quad (17)$$

By rewriting (10) and (16), upon using for brevity the quantities A_m, B_{mn}, C_m, D_{mn},

$$A_m = j\omega\mu r \qquad B_{mn} = j\omega\mu H_{mn}$$
$$C_m = j\frac{\beta_m^2 r}{\omega\mu} \qquad D_{mn} = j\left(\frac{\beta_n^2}{\omega\mu}H_{mn} + \frac{f_{mn}}{\omega\mu}\right) \quad (18)$$

we get the system of equations (3, 4). This gives the differential relationship between voltages and currents expressing the amplitude of the electric and magnetic field, respectively, along the bend. The solution of the telegrapher's equations (3, 4) is obtained in the following manner.

B. Solution of the Telegrapher's equations

By introducing an appropriate matrix $[\tau]$ the generalised telegrapher equations (3, 4) are rewritten in the following way

$$\partial_\varphi \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix} = [\tau] \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix}_0 \quad (19)$$

For a bend of angle θ and constant radius the matrix $[\tau]$ is a constant, thus allowing the solution of the above system as

$$\partial_\varphi \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix} = e^{\theta[\tau]} \begin{bmatrix} \mathbf{v} \\ \mathbf{i} \end{bmatrix}_0 \quad (20)$$

where the subscript 0 in the r.h.s. term stand for the voltage and current distribution at $\theta = 0$. The matrix $e^{\theta[\tau]}$, i.e. the transmission matrix of the bend, is computed by using the formula:

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = e^{\theta[\tau]} = \sum_{n=0,1}^{\infty} \frac{(\theta[\tau])^n}{n!} \quad (21)$$

In order to overcome any numerical instability arising from the use of the transmission matrix with several modes below cut-off, it is expedient to compute the solution for a small angle $\Delta\varphi = \frac{\theta}{2^M}$ and then to transform the transmission matrix into the scattering matrix. The generalised scattering matrix of the entire bend is obtained by calling M times the routine providing the scattering matrix of double a given angle.

This approach, apart for being numerically efficient and very stable, is also well suited to accounting for matching elements inside the bend. In fact, it is easy to rigorously analyze the effects of a discontinuity by describing the latter in terms of its accessible modes. Note that, by numerically solving (19), the local mode technique also allow to consider bends with varying angle of curvature, as well as serpentines etc.

III. Wide-Band Matched Bends

It is convenient to distinguish between E- and H- plane bends, since their design is fairly different.

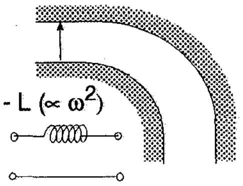

Fig. 2. E-plane bend with equivalent circuit shown in the inset.

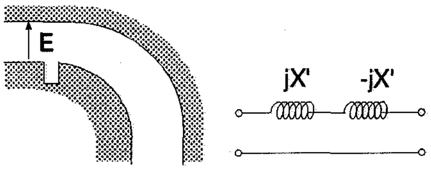

Fig. 3. Wide-band matched E-plane bend, by using a stub as series inductance. The resulting equivalent circuit is shown in the inset.

A. E-plane WBM bend

It was suggested in [4] to match an E-plane bend by means of an E-plane stub. The matching mechanism is readily understood by considering that an E-plane bend behaves as a series negative inductor [5], see Fig. 2. Thus by adding in series to this a positive inductance, as shown in the inset of Fig. 3, we realize an element with the following ABCD matrix:

$$\begin{bmatrix} 1 & j(X' - X) \\ 0 & 1 \end{bmatrix} \quad (22)$$

It is apparent that, by proper selecting the frequency dependence of the X' one can compensate the frequency dependence of X over a fairly large bandwidth. A suitable ME to this end is a short (inductive) E-plane stub, such as depicted in Fig. 3, which allows to realize E-plane matched bends over the whole X-band ($8.2 - 12.4\ GHz$).

B. H-plane WBM bend

The H-plane bend corresponds to a parallel inductor [5] and is slightly more difficult to match, two solutions being feasible. One design compensates the bend by using an E-plane folded stub [3], that is a series capacitance, which allows to obtain the desired overall frequency-dependent reactance. The other approach compensates the parallel inductance of the H-plane bend by using a shunt capacitance as shown in the inset of Fig. 4. The latter design, although mechanically simpler, offers fewer degrees of freedom in achieving the sought frequency-dependent reactance.

IV. Experimental and numerical results

A FORTRAN computer code, based on the local modes approach, has been developed and tested against pub-

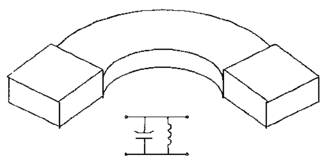

Fig. 4. A matched H-plane bend in rectangular waveguide. An E-plane step, corresponding to a shunt capacitance, is placed before the bend, which corresponds to a parallel inductance

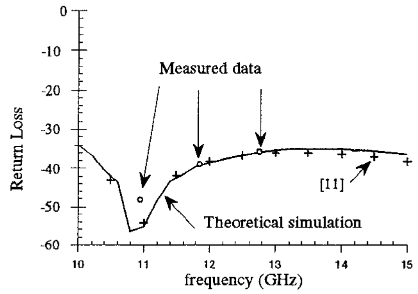

Fig. 5. Return loss of a circular 90° H-plane bend in rectangular waveguide (guide width $a = 19.05$ mm, guide height $b = 9.530$ mm, mean radius is $R = 15.6$ mm). Continuous curve refers to our numerical simulation, crosses refer to the numerical simulation of [11]; circles are experimental data.

lished data of an unmatched bend. In Fig. 5, 6 we report the comparison of our simulations (continuous line) with the theoretical data of [11] (crosses) relative to the method presented in [8]. In Fig. 5 theoretical results have been compared with measured data showing an excellent agreement. From extensive numerical tests it has been found that it is normally sufficient to consider the coupling between 4- 5 modes to obtain quite accurate results. In this case, the computation takes few seconds for each frequency point on a PC.

From Fig. 6 it is noted that at the frequency of 12.5 GHz, for a bend of 120° and for a radius of 10 mm, we have a very low reflection coefficient. The knowledge of this angle is important since it allows designing the layout of rather complicated beam-forming networks without return losses due to bends. However, this circumstance is useless when the curve must be of a given angle; in addition, wherever a larger bandwidth is required, it is necessary to employ ME and the design described in the previous section.

V. Conclusions

Wide-band matched (WBM) waveguide bends with small radius of curvature have been realized by placing matching elements (ME) inside the curve. The design of WBM E-plane bend has been accomplished by using an E- plane stub at the beginning of the curve as ME. For the H-plane bend two different solutions have been proposed. The matched bend is designed by using an efficient computer code based on the local modes approach. Theoretical simulations have been compared with experimental results showing very good accuracy.

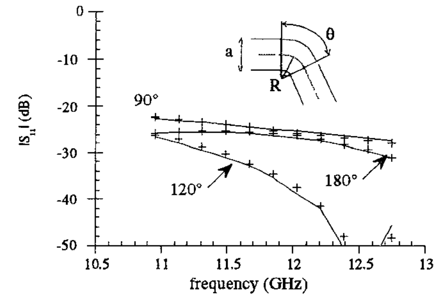

Fig. 6. Return loss of a circular H-plane bend in rectangular waveguide (guide width $a = 19.05$ mm, guide height $b = 9.53$ mm). The mean radius is $R = 10$ mm, and the bend angles are indicated in the graph. The continuous curves refer to our numerical simulation, while crosses refer to the numerical simulation of [11].

References

[1] H. H. Meinel, "Millimeter-wave technology advances since 1985 and future trends," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-39, pp. 759-767, May 1991.

[2] F. Alessandri, M. Mongiardo, R. Sorrentino, "Computer-Aided Design of Beam Forming Networks for Modern Satellite Antennas", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-40, No. 6, June 1992.

[3] F. Alessandri, M. Mongiardo, R. Ravanelli, "A Compact Wide-Band Variable Phase Shifter for Reconfigurable Satellite Beam Forming Networks," *Proc. 23rd European Microwave Conf.*, pag.556-557, 1993.

[4] F. C. de Ronde, "Full-band matching of waveguide discontinuities," *MTT-S International Microwave Symposium*, Palo Alto, California, 1966.

[5] N. Marcuvitz, *Waveguide Handbook*, McGraw-Hill, New York, 1951.

[6] M. Mongiardo, R. Sorrentino, "Efficient and versatile analysis of microwave structures by combined mode matching and finite difference methods," *IEEE Microwave and Guided Wave Letters*, Aug. 1993.

[7] J.M. Reiter, F. Arndt, "A full-wave boundary contour mode-matching method (BCMM) for the rigorous CAD of single and cascaded optimized H- plane and E-plane bends," *Proc. IEEE MTT-S Digest*, pp. 1021-1024, 1994.

[8] L. Accatino, G. Bertin, "Modal analysis of curved waveguides," *Proc. 20th Europ. Microwave Conf.*, Budapest, pp. 1246-1250, Sept. 1990.

[9] A. Weisshaar, S.M. Goodnick, V.K. Tripathi, "A rigorous method of moments solution for curved waveguide bends and its applications," *Proc IEEE MTT-S Digest*, pp. 975-978, 1992.

[10] V. V. Shevchenko, *Continuous Transition in Open Waveguides*, Golem Press, Boulder, Colorado, 1971

[11] L. Accatino, Private communication.